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COMMENT

A fermionic loop space for QCD

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Abstract. We propose a generalisation of the usual bosonic loop-space formulation of gauge theory in the case of fermionic interacting matter.

One of the most interesting problems in particle physics is that of understanding QCD in terms of colour singlet fields [1, 2].

Our aim in this comment is to propose a generalisation of the usual bosonic loop space formulation for gauge theory in the case of fermionic interacting matter [3] (quantum chromodynamics).

Let us start our analysis by considering the QCD Euclidean partition functional with the fermionic quark degrees integrated out:

$$Z^{\mathrm{QCD}} = \int DA_\mu \exp(-S[A_\mu]) \operatorname{Det}[\not{D}(A_\mu)] \tag{1}$$

where $S[A_\mu]$ denotes the Yang-Mills action and $\not{D}(A_\mu) = \gamma_\mu(\partial_\mu + A_\mu)$ is the Euclidean Dirac operator in the presence of the external Yang-Mills field $A_\mu(x)$.

By using the proper-time definition for the above-mentioned functional determinant, we consider the formal relationship for its modulus [4]

$$\begin{aligned} \tfrac{1}{2}\log \operatorname{Det}[\not{D}(A_\mu)]| &= |\log[\operatorname{Det}(\tfrac{1}{2}[\mathrm{D}(A_\mu)\mathrm{D}^*(A_\mu) + \mathrm{D}^*(A_\mu)\mathrm{D}(A_\mu)])]^{1/2}| \\ &= -\int_0^\infty \frac{\mathrm{d}\tau}{\tau} \operatorname{Tr}[\exp(-\mathbb{H}(A_\mu)\tau)] \end{aligned} \tag{2}$$

where we have introduced the (Euclidean) Hamiltonian

$$\mathbb{H}(A_\mu) = [\tfrac{1}{2}(\mathrm{D}(A_\mu)\mathrm{D}^*(A_\mu) + \mathrm{D}^*(A_\mu)\mathrm{D}(A_\mu))]^{1/2}. \tag{3}$$

In the loop approach to QCD the next step is to write the propagator $\operatorname{Tr}[\exp(-\mathbb{H}(A_\mu)\tau)]$ as a kind of 'continuous' sum of closed trajectories [2]. At this point we introduce our suggestion: since the Hamiltonian $\mathbb{H}(A_\mu)$ corresponds to a particle possessing Lorentz and colour spin (interacting with the external Yang-Mills fields) the closed trajectories entering into the Feynman path-integral expression for $\operatorname{Tr}[\exp(-\mathbb{H}(A_\mu)\tau)]$ should reveal in an explicit way their fermionic particle dynamical degrees of freedom.

A natural framework for analysing this case is pseudoclassical mechanics (for a general reference see, e.g., [5]) where the worldline of a spinning coloured particle is described by the usual vector position $X^{\mu}(\xi)$ added to a set of Grassmann complex variables $\{\theta_l(\xi), \theta_l^*(\xi)\}$ associated with the particle colour charges [5] and another set of real $\psi_{\mu}(\xi)$ Grassmann variables corresponding to the Lorentz spin [6].

In the simplest Abelian case, the above path integral expression was proposed by Rumpf [7] and given explicitly by the following expression:

$$\begin{aligned}\operatorname{Tr}[\exp(-\mathbb{H}(A_{\mu}))] &= \int \mathrm{d}^D X \int_{X_{\mu}(0)=0=X_{\mu}(\tau)=X} \mathrm{D}(X_{\mu}(\xi)) \operatorname*{Tr}_{\text{Dirac}} \\ &\quad \times \int_{\psi_{\mu}(0)=\psi_{\mu}(\tau)} \mathrm{D}(\psi_{\mu}(\xi)) \exp[-L(X_{\mu}(\xi), \psi_{\mu}(\xi) A_{\mu}(\xi))] \end{aligned} \tag{4}$$

where the pseudoclassical Lagrangian $L(X_{\mu}(\xi), \psi_{\mu}(\xi), A_{\mu}(\xi))$ is given by [6]:

$$L(X_{\mu})(\xi), \psi_{\mu}(\xi), A_{\mu}(\xi)) = \tfrac{1}{4}(\dot{X}_{\mu})^2 - \tfrac{1}{4}\mathrm{i}\psi_{\mu}\dot{\psi}_{\nu} + A_{\mu}(X)\dot{X}_{\mu} + \tfrac{1}{4}\mathrm{i}[\psi_{\mu}, \psi_{\nu}]F_{\mu\nu}(X(\xi)). \tag{5}$$

In the non-Abelian case we propose to consider the analogues of (4) and (5), the coloured version of our previous pure coloured path integral (see equation (2) of [5]):

$$\begin{aligned}\operatorname{Tr}[\exp(-\mathbb{H}(A_{\mu})\tau)] &= \int \mathrm{d}X^D \int_{X_{\mu}(0)=X_{\mu}(\tau)=X} \mathrm{D}[X_{\mu}(\xi)] \operatorname*{Tr}_{\text{Dirac}} \\ &\quad \times \int_{\psi_{\mu}(0)=\psi_{\mu}(\xi)} \mathrm{D}(\psi_{\mu}(\xi)) \int \mathrm{D}(\theta(\xi)) D(\theta^*(\xi)) \theta_i(0)\theta_i^*(\tau) \\ &\quad \times \exp[-L(X_{\mu}(\xi), \psi_{\mu}(\xi), \theta(\xi), \theta^*(\xi), A_{\mu}(X))] \end{aligned} \tag{6}$$

where our proposed pseudoclassical gauge-invariant Lagrangian for a spinning coloured particle is given by

$$\begin{aligned} L(X_{\mu}(\xi), \theta(\xi), \theta^*(\xi), \psi_{\mu}(\xi), A_{\mu}(X)) &= \tfrac{1}{2}(\dot{X}_{\mu})^2 + \tfrac{1}{4}\mathrm{i}\dot{\psi}_{\mu}\psi_{\nu} + \tfrac{1}{2}\mathrm{i}\left(\sum_{i=1}^{N} (\theta_i^*\dot{\theta}_i - \dot{\theta}_i^*\theta_i)(\xi)\right) - g(\theta_l^*(\lambda_i)_{lk}\theta_k)(\xi) \\ &\quad \times A_{\mu}^i(X(\xi))\dot{X}^{\mu}(\xi) + \tfrac{1}{4}\mathrm{i}[\psi_{\mu}, \psi_{\nu}](\xi)(\theta_l^*(\lambda_i)_{lk}\theta_k)(\xi)F_{\mu\nu}^i(X(\xi)). \end{aligned} \tag{7}$$

By exactly integrating out the colour Grassmannian variables in (12) as in [5], we can see the natural appearance of the fermionic Wilson loop factor considered in [8] for quantum chromodynamics:

$$W[X_{\mu}^{(\mathrm{F})}(S, \theta)] = \operatorname*{Tr}_{\text{colour}} P \exp\left(\int_0^1 \mathrm{d}S \int \mathrm{d}\theta\, A_{\mu}(X_{\mu}^{\mathrm{F}}(S, \theta))\, \mathrm{D}X_{\mu}^{\mathrm{F}}(S, \theta)\right) \tag{8}$$

where we have used a super-loop notation to write the Yang–Mills interacting term in (7) in a compact form.

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